INTERACTION OF TWO ISOTROPIC TURBULENT FIELDS WITHOUT SHEAR

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Results of experiments conducted under simple conditions have great significance for the complex problem of turbulent flow which still does not have a satisfactory solution. Thus, the turbulent flow is appreciably simplified if even one term in the energy equation for turbulent fluctuations is absent: convection, diffusion, production, or dissipation. One of the simplest is the flow past a uniform grid where there is no diffusion or production of turbulent energy and moreover all necessary conditions for isotropic fluctuations could be satisfied.

Flow without diffusion of turbulent energy was realized, e.g., in [1]. Flow without shear in the mean flow, and consequently, without turbulent energy production was first studied experimentally in [2]. These conditions also exist in axisymmetric hydrodynamic wake with zero excess momentum and an experimental result on this problem is given in [3]. A grid vibrating perpendicular to its plane in a reservoir [4] has a very simple turbulent field. In this case the mean velocity is zero and there is a region in which only diffusion and disipation of energy are present.

A description along with certain results of studies on one more simple turbulent flow without a shear in the mean velocity and in which convection and diffusion are one-dimensional are presented below. It appears to be most interesting for the analysis of problems on interactions of turbulent fields with each other, e.g., problems on the evolution of boundary layer, jet, or wake in turbulent free stream. With certain additional modifications this flow also makes it possible to effectively analyze questions on the effect of density stratification on turbulence.

The basis for the present method to realize shearless turbulence is the experimental fact [5] that the pressure drop in uniform hydrodynamic grid which depends in general on the parameters $Re = U_0M/\nu$ and M/D, where U_0 is the free stream velocity, D is the diameter of the rods in the grid, M is the distance between axes of the rods, and ν is the kinematic viscosity of the fluid, ceases to depend on Reynolds number Re if it is sufficiently large. For such an asymptotic condition for Re it is possible to change the diameter and the pitch of rods in separate segments of the grid, maintaining the condition M/D = const, and this does not lead to the appearance of transverse pressure gradients and gradients in mean velocity in the flow behind the grid. At the same time it is possible to vary within a wide range the nonuniformity in the intensity across the flow, the characteristic scale and other statistical characteristics of fluctuations, i.e., it is possible to create a specific stratification of turbulence characteristics. If, in addition, some of the rods are heated, then there will also be a stratification in density: stable or unstable, vertical or horizontal, continuous or in steps, depending on how heating is carried out.

Results of studies on isothermal flow past a grid consisting of two parts (Fig. 1) are presented in this paper. The upper half of the grid had rods with diameter D_1 = 1 mm and the distance between them M_1 = 5.5 mm, the lower half had rods with $D_2 = 5D_1$ and $M_2 = 5M_1$. Horizontal and vertical rods were located in different planes (the so-called biplane configuration of the grid). The grid was placed in the test section of a low-turbulence wind tunnel with a characteristic transverse dimension of 40 cm and a length of 4 m. Tests were conducted at a free stream velocity of 15 m/sec and the air temperature was 19°C. Additional contraction of the flow behind the grid was intended to improve the level of isotropy of fluctuations (see details in [6]), so that the mean velocity in the measurement section $U = 1.56U_0$.

The connection of the two halves of the grid required some care because these could lead to local distortion of the mean velocity field. As a result of testing a number of variants it was found that good results were obtained by a certain reduction in the diameter of the

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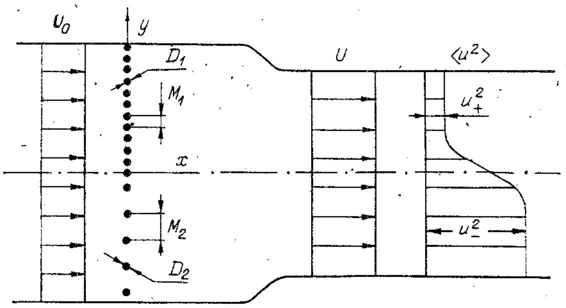

Fig. 1

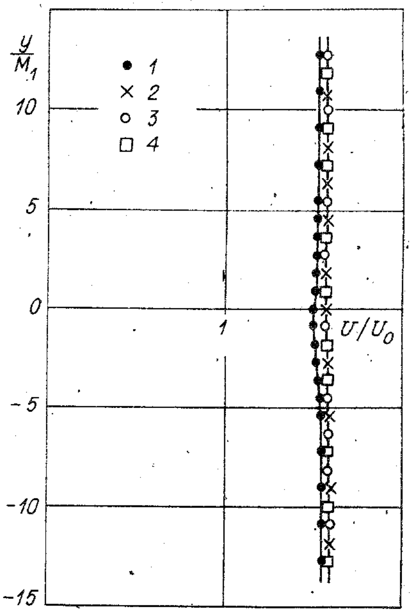

Fig. 2

horizontal rod D_2 closest to the connection line while simultaneously reducing the first gap M_2 between the horizontal rods. The coordinate y is measured vertically above the juncture line, x is downstream along the flow. The flow is completely uniform along the z axis in the statistical sense, i.e., all its statistical characteristics are invariant with respect to displacement along this coordinate.

The measurement of velocity was carried out with standard DISA hot-wire anemometer equipment. Histomat-S of the firm Intertechnique was used for the statistical analysis of the signals from the anemometer. Mean velocities were also measured with Pitot tube. The error in the experimental data presented here was estimated by the variance not exceeding 2% for the mean flow and 4% for the intensity of fluctuations. These estimates were obtained on the basis of results from repeated measurements at different points in the flow at the same flow conditions.

The degree of uniformity in mean velocity U along y and x in the realized flow is illustrated by experimental data for velocity profiles U(y) for a number of fixed values of x/M_1 as shown in Fig. 2 (the points 1-4 correspond to x/M_1 = 40, 80, 160, and 240). It is possible to observe that conditions for zero shear in the flow are quite accurately met.

Profiles of the dispersion of fluctuations in the longitudinal velocity component $\langle u^2 \rangle$ are shown in Fig. 3 (angular brackets denote the averaging operator, the points 1,...,6 correspond to x/M_1 = 40,...,240, with an interval of 40). They show that along with the production downstream due to viscous effects, the initial nonuniformity in the direction of $\langle u^2 \rangle$ along y is gradually smoothened out because of turbulent diffusion. Here the mean point y_0 of the profile $\langle u^2 \rangle$, i.e., that value of y where $\langle u^2 \rangle = u_0^2 = (u_+^2 + u_-^2)/2$ (the quantities u_+^2 and u_-^2 are explained in Fig. 1), is more and more displaced towards the direction of lower turbulence intensity with an increase in x. This interesting feature of the diffusion process should be taken into account in its mathematical modeling.

It is interesting to consider whether it is possible or not to express the $\langle u^2 \rangle$-profile obtained at various values of x/M_1, in a universal form, by making appropriate use of the transformation in stretching and displacement along y. This question of affine transformation of profiles is closely associated with the existence of similarity solutions of the equations for the quantities $e = (u^2 + v^2 + w^2)/2$ (v and w are the projections of fluctuating velocity component on y and z axes, respectively), since the equation for $\langle e \rangle$ and $\langle u^2 \rangle$ are similar in form and in mathematical modeling of turbulence the equation for $\langle e \rangle$ is mainly used.

Since all statistical characteristics of the given flow are uniform along z, the mean velocity lies along x and y, and since the flow is stationary in the statistical sense, the energy equation for turbulence $\langle e \rangle$ [5] can be written in the form

$$U \frac{\partial \langle e \rangle}{\partial x} = -\frac{\partial}{\partial x} \left\langle u \left(\frac{p}{\rho} + e \right) \right\rangle - \frac{\partial}{\partial y} \left\langle v \left(\frac{p}{\rho} + e \right) \right\rangle + \varepsilon_1 - \varepsilon_2,$$

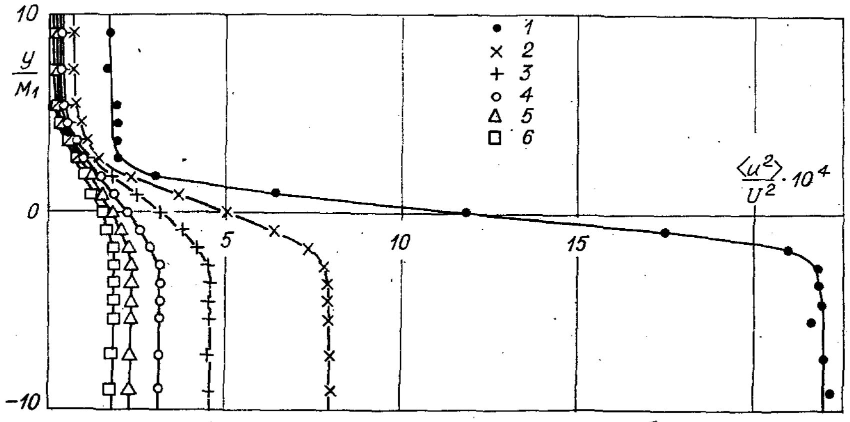

Fig. 3

$$\varepsilon_1 = \nu\left\langle \frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 e}{\partial x^2} + \frac{\partial^2 v^2}{\partial y^2} + \frac{\partial^2 e}{\partial y^2} + 2\,\frac{\partial^2 (uv)}{\partial x \partial y}\right\rangle, \quad (1)$$
$$\varepsilon_2 = \nu\left\langle \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} + \frac{\partial u_j}{\partial x_i}\frac{\partial u_j}{\partial x_i}\right\rangle,$$

where p is the pressure fluctuation; ρ is the fluid density; i, j = 1, 2, 3; summation is carried out with repeated indices; in order to shorten the expression for the last relation, the notation for the system of coordinates is changed: $x = x_1$, $y = x_2$, etc.

Further simplifications of (1) are possible for certain asymptotic conditions when individual terms happen to be an order of magnitude smaller than others and they could be neglected. For example, at fairly large values of Re and for sufficiently large distances from the grid

$$\langle u(p/\rho + e)\rangle \ll U\langle e\rangle,$$

so that (1) takes the form

$$U\frac{\partial \langle e\rangle}{\partial x} \simeq -\frac{\partial}{\partial y}\left\langle v\left(\frac{p}{\rho} + e\right)\right\rangle - \varepsilon, \quad (2)$$

where $\varepsilon = -\varepsilon_1 + \varepsilon_2$. The left-hand side of this equation describes convective transfer, the first term on the right-hand side denotes diffusion. The last term is basically determined by energy dissipation. Under certain conditions it is also possible to simplify the discussion term and the expression for ε by throwing out higher-order terms. However, additional experimental data are necessary for corresponding evaluations and to analyze the problem on the existence of similarity solutions there is no need for further

$$\langle e\rangle = U_*^2 f_1(\eta), \quad \left\langle v\left(\frac{p}{\rho} + e\right)\right\rangle = U_*^3 f_2(\eta), \quad \varepsilon = \frac{U_*^3}{l_*} f_3(\eta); \quad (3)$$

$$\eta = (y - y_0)/l_* \quad (4)$$

so that

$$U_* = c_1[(x - x_0)/c_2]^m, \quad l_* = c_3[(x - x_0)/c_2]^n, \quad (5)$$
$$y_0 = c_4 l_* + c_5, \quad n - m = 1,$$

c_k, x_0, m, and n are quantities that do not depend on x and y. It is interesting to note the presence in Eq. (4) of the term $y_0(x)$ whose significance was explained above.

The above relations do not allow an independent determination of the indices m and n. In order to achieve this it is necessary to obtain additional relations. Here the following assumption is made

$$\mathrm{Re}_* = U_* l_*/\nu = \mathrm{const}.$$

It is based on the existing experimental data in the literature on two-dimensional free turbulent flows (mixing layers, jets, and wakes) and finds indirect confirmation in experimental data pertaining to the present problem. It follows from this assumption that m + n = 0. This predetermined the exponent in (5): m = −1/2, n = 1/2. In order to complete the analysis of

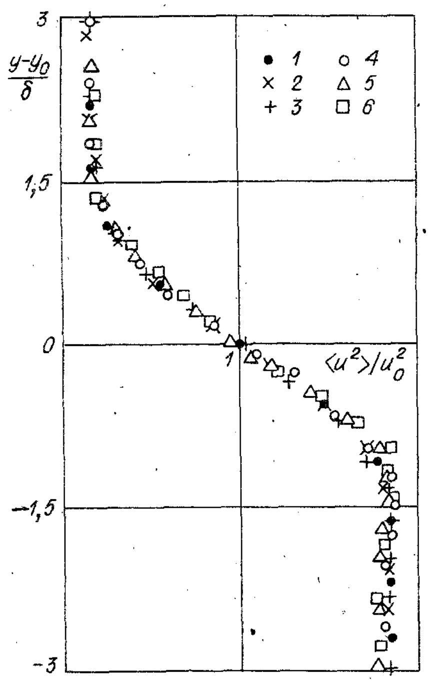

Fig. 4

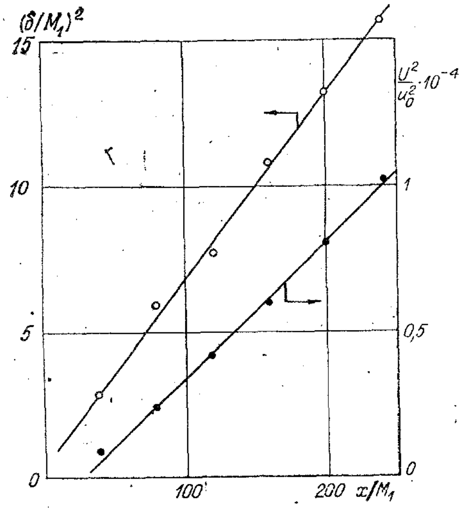

Fig. 5

similarity solutions to Eq. (2) it is necessary to determine the nature of functions f_1, f_2, and f_3. But this requires the introduction of closure relations for Eq. (2) which is out of scope of the present study.

The experimental data on the present flow are shown in Fig. 4 in similarity variables for $\langle u^2\rangle$. u_0 determined above is used for U_*, l_* is the characteristic half-width of the profile $\langle u^2\rangle$, i.e., the difference $\delta = y_2(x) - y_1(x)$, where y_1 and y_2 are determined by the conditions

$$\langle u^2\rangle = (u_+^2 + 3u_-^2)/4 \text{ at } y = y_1, \quad \langle u^2\rangle = (3u_+^2 + u_-^2)/4 \text{ at } y = y_2.$$

It is possible to observe that the experimental data do not contradict the assumption on similarity profiles $\langle u^2\rangle$. It is worth noting the absence of symmetry in the profiles of $\langle u^2\rangle$, a fact not often encountered in diffusion process. It should be taken into account in mathematical modeling.

The dependence of $(U/u_0)^2$ and $(\delta/M_1)^2$ on x/M_1 is shown in Fig. 5. The fact that experimental points in such a representation are concentrated well along straight lines confirms the nature of dependence of u_0 and δ on x, in particular, the above-mentioned power relation. But there is also one disagreement with analytical results of similarity solutions: the value of x_0 determined as the point of intersection of the straight lines in Fig. 5 with the x axis happen to be different for the functions $u_0(x)$ and $\delta(x)$ in the experimental data whereas they should be the same according to the analysis.

The following empirical relations have been obtained using the method of least squares:

$$u_0/U = 0.15(x/M_1 - 28.3)^{-0.5}, \quad \delta/M_1 = 0.25(x/M_1 + 5.97)^{0.5}, \quad y_0/M_1 = 0.85\delta/M_1 - 1.46. \tag{6}$$

The question of their universality for different grids remains open until suitable experiments are conducted. This question is of great independent interest since it is closely associated with the characteristics of relaxation processes in turbulent flows. At present there is an accumulation of sufficiently large experimental data (see, e.g., [7]) on the fact that turbulence has surprisingly long memory of the conditions of its formation, a fact that considerably complicates its mathematical modeling. It is not excluded that even in the present case, by some changes in the normalizing scales U and M_1 in Eq. (6), it may be possible to obtain universality of these relations for different grids.

Typical results from the measurements of one-dimensional probability density distributions for the fluctuations of longitudinal velocity component $\varphi(u/\sigma)$, where $\sigma = \sqrt{\langle u^2\rangle}$ are shown in Fig. 6. They correspond to the section $x/M_1 = 160$. The curve 1 corresponds to such large values of $\pm y/M_1$ where turbulent field generated by different parts of the grid do not interact

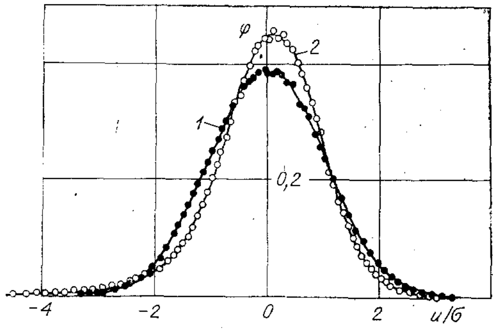

Fig. 6

with each other and are isotropic. Here the one-dimensional probability density distribution is Gaussian.

The curve 2 in Fig. 6 is obtained for $y/M_1 = 3.64$ inside the mixing zone. It strongly differs from Gaussian distribution. Its skew is −0.66, excess is 2.03 whereas for Gaussian distribution they are zero. Such a character of φ in the mixing zone is associated with the peculiar intermittency of the flow when large and stronger eddies produced by the lower part of the grid alternate with smaller and weaker eddies generated by the upper half of the grid. The fact that intermittency of eddies of different nature leads to a difference in the distribution from Gaussian was demonstrated in [8]. This strongly complicates the mathematical modeling of turbulent flows. In order to describe them it is necessary to take up not only arithmetical but also logical summation of fluctuations from various sources. The intermittency of fluctuations expressed in probability distributions at such large distances from the grid indicates that turbulent eddies retain their individuality very long which is associated with the memory of turbulence about the conditions of their formation.

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